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Flat photonic bands in two-dimensional photonic crystals with kagome lattices

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Abstract

We theoretically demonstrate flat photonic bands in two-dimensional photonic crystals with kagome lattices. In such photonic crystals composed of circular rods, electromagnetic waves localizing at the rods form flat photonic bands. The flat photonic bands are important for the omnidirectional lasing and the enhancement of the omnidirectional electromagnetic interaction of materials. Dependences of flat photonic bands on the dielectric constant and radii of rods are discussed.

1. Introduction

Recently, dielectric periodic structures of the order of electromagnetic wavelength have attracted much attention as photonic crystals, from both theoretical and practical viewpoints [1–9]. Photonic crystals have interesting optical properties such as photonic band gaps in which light of certain frequency ranges cannot propagate. In photonic crystals with omnidirectional photonic band gaps, for example, point and linear defects cause light localization and guiding, respectively.

Moreover, photonic crystals have high dispersion relations between frequencies and wave vectors in photonic band structures, and the group velocities of electromagnetic waves become zero at photonic band edges. The physical meaning of zero group velocities is that several scattering waves form standing waves in photonic crystals and, therefore, we can obtain coherent waves, i.e., lasing at the photonic band edges [10, 11]. Moreover, the enhancement of electromagnetic interaction of materials can be expected at the photonic band edges because of the long interaction time.

In conventional photonic crystals, however, such effects for electromagnetic waves with certain frequencies are obtained only in certain directions, since the photonic band edges depend on the directions of the electromagnetic waves. For further applications, the lasing and the enhancement of the electromagnetic interaction of materials are necessary in any direction, which can be achieved by flat photonic bands. For the effective enhancement of the electromagnetic interaction of the materials, in particular, it is appropriate for electromagnetic fields to localize at the materials. Although group velocities of electromagnetic waves become very slow at high frequencies in conventional photonic crystals composed of materials with high dielectric constants, the electromagnetic waves do not always localize at the materials. Therefore, it is necessary that in photonic crystals, electromagnetic waves satisfying flat photonic bands localize at the materials, regardless of the directions of the electromagnetic waves.

In the solid state, on the other hand, kagome lattices are well known as structures with flat bands [12, 13]. In such lattices, singular magnetic properties such as flat-band ferromagnetism can be expected, since the electron correlation exceeds kinetic energies at the flat bands because of the zero group velocities of electrons. In the tight-binding model, the flat bands are formed by electrons localizing at atoms which interact with electrons at the nearest-neighbour atoms. However, the flatness of the flat bands deteriorates due to interactions between electrons at second-nearest-neighbour atoms.

Recently, it has been reported that flat photonic bands are formed in photonic band gaps by periodic point defects in two-dimensional photonic crystals [14]. That is, photonic bands with very small gradients are formed by electromagnetic fields localizing at point defects. Because of greater simplicity of fabrication, however, it is more desirable that flat photonic bands can be obtained in two-dimensional photonic crystals without any defects. Therefore, we propose the use of two-dimensional photonic crystals with kagome lattices to obtain flat photonic bands. The photonic crystals are assumed to be composed of circular rods. In the transverse magnetic (TM) mode, electromagnetic waves tend to localize at the rods and, therefore, it is appropriate to apply the tight-binding model to the TM mode although there also exist photonic bands in which electromagnetic waves do not localize at rods [15]. Therefore, we consider only the TM mode. Although it is well known that photonic crystals composed of rods have photonic band gaps in the TM mode, we do not consider the existence of photonic band gaps in this paper. In photonic band structures, there exist quasi flat photonic bands formed by electromagnetic waves localizing at rods. We investigate photonic band structures calculated by the plane wave method based on the tight-binding model. We theoretically discuss the dependences of quasi flat photonic bands on the dielectric constant and the radii of rods.

2. Theory

Figure 1 shows a two-dimensional photonic crystal with kagome lattices composed of circular rods.

In the TM mode, the electric field $E_z(\boldsymbol{r})$ satisfies the following equation:

$$-\frac{1}{\epsilon(\boldsymbol{r})}\nabla^2 E_z(\boldsymbol{r}) = \left(\frac{\omega}{c}\right)^2 E_z(\boldsymbol{r}), \tag{1}$$

where $\epsilon(\boldsymbol{r})$, ω and c are the dielectric constant, the frequency and the speed of light in vacuum, respectively.

2.1. Tight-binding method

We focus our attention on photonic band structures formed by electric fields localizing at rods by the tight-binding method in analogy with quantum mechanics. Calculations using

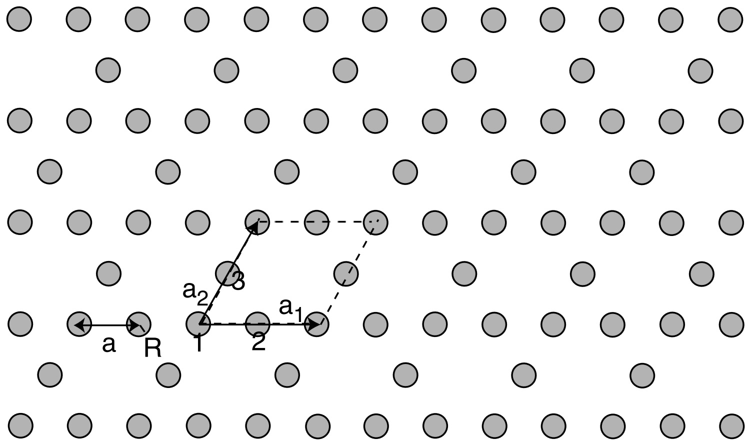

Figure 1. Two-dimensional photonic crystal with kagome lattices composed of circular rods. The shaded circle indicates the rod. R is the radius of the rod, and a is the distance between the nearest-neighbour rods. The region embedded by dotted lines is the unit cell, and $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$ are basic vectors of kagome lattices. 1, 2, and 3 indicate sites of rods.

the tight-binding method have already been reported [16, 17]. Unlike the Hamiltonian in Schrödinger equations, the operator $-(1/\epsilon(\boldsymbol{r}))\nabla^2$ in equation (1) is not a Hermitian operator. Therefore, instead of equation (1), we consider the following equation:

$$-\frac{1}{\sqrt{\epsilon(\boldsymbol{r})}}\nabla^2\frac{1}{\sqrt{\epsilon(\boldsymbol{r})}}F_z(\boldsymbol{r}) = \left(\frac{\omega}{c}\right)^2 F_z(\boldsymbol{r}), \tag{2}$$

where $F_z(\boldsymbol{r}) = \sqrt{\epsilon(\boldsymbol{r})}E_z(\boldsymbol{r})$. The operator $\Theta(\boldsymbol{r}) = -(1/\sqrt{\epsilon(\boldsymbol{r})})\nabla^2(1/\sqrt{\epsilon(\boldsymbol{r})})$ is the Hermitian operator [15].

The field $\Phi_{i=1,2,3}(\boldsymbol{r})$ is assumed to localize at the site i (=1–3) and satisfy normalized orthogonal relations. According to Bloch's theorem, the field $F_z(\boldsymbol{r})$ satisfies

$$F_z(\boldsymbol{r}) = \sum_{\boldsymbol{R}} \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{R}} \sum_{i=1}^{3} c_i \Phi_i(\boldsymbol{r}-\boldsymbol{R}), \tag{3}$$

where c_i is the coefficient of $\Phi_i(\boldsymbol{r})$ at the site i, $\boldsymbol{R}$ the lattice vector of kagome lattices, and $\boldsymbol{k}$ the wave vector. The interaction between the fields localizing at neighbouring rods is as follows:

$$\langle\Phi_i(\boldsymbol{r})|\Theta(\boldsymbol{r})|\Phi_j(\boldsymbol{r})\rangle = (\langle\Phi_j(\boldsymbol{r})|\Theta(\boldsymbol{r})|\Phi_i(\boldsymbol{r})\rangle)^*$$

$$= \int \Phi_i^*(\boldsymbol{r})\Theta(\boldsymbol{r})\Phi_j(\boldsymbol{r})\,\mathrm{d}\boldsymbol{r} = \begin{cases} \alpha & (i=j) \\ -\beta & \text{(nearest-neighbour interaction)} \\ -\beta' & \text{(second-nearest-neighbour interaction).} \end{cases} \tag{4}$$

In the tight-binding model, the following equation is obtained [13]:

$$\begin{bmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \alpha \end{bmatrix} \boldsymbol{c} - \beta \begin{bmatrix} 0 & 1+\mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{a}_1} & 1+\mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{a}_2} \\ 1+\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{a}_1} & 0 & 1+\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot(\boldsymbol{a}_1-\boldsymbol{a}_2)} \\ 1+\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{a}_2} & 1+\mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot(\boldsymbol{a}_1-\boldsymbol{a}_2)} & 0 \end{bmatrix} \boldsymbol{c}$$

$$- \beta' \begin{bmatrix} 0 & \mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{a}_2}+\mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot(\boldsymbol{a}_1-\boldsymbol{a}_2)} & \mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{a}_1}+\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot(\boldsymbol{a}_1-\boldsymbol{a}_2)} \\ \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{a}_2}+\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot(\boldsymbol{a}_1-\boldsymbol{a}_2)} & 0 & \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{a}_1}+\mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{a}_2} \\ \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{a}_1}+\mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot(\boldsymbol{a}_1-\boldsymbol{a}_2)} & \mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{a}_1}+\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{a}_2} & 0 \end{bmatrix} \boldsymbol{c} = \left(\frac{\omega}{c}\right)^2 \boldsymbol{c}, \quad (5)$$

where $\boldsymbol{c} = [c_1, c_2, c_3]^{\mathrm{T}}$. Photonic band structures obtained by solving equation (5) can be seen as those formed by electric fields localizing at rods, since $F_z(\boldsymbol{r})$ is proportional to $E_z(\boldsymbol{r})$.

2.2. Plane wave method

We calculate general photonic band structures by the plane wave method [7]. The dielectric constant is periodic with respect to $\boldsymbol{R}$ generated by the primitive translation and may be expanded in a Fourier series on $\boldsymbol{G}$, the reciprocal lattice vector,

$$\epsilon(\boldsymbol{r}) = \sum_{\boldsymbol{G}} \epsilon(\boldsymbol{G}) \exp(\mathrm{i}\boldsymbol{G}\cdot\boldsymbol{r}). \quad (6)$$

Using Bloch's theorem, we may expand the electric field as

$$E_z(\boldsymbol{r}) = \sum_{\boldsymbol{G}} E(\boldsymbol{G}) \exp\{\mathrm{i}(\boldsymbol{k}+\boldsymbol{G})\cdot\boldsymbol{r}\}, \quad (7)$$

where $\boldsymbol{k}$ is the wave vector. By substituting equations (6) and (7) into (1), the following equation is obtained:

$$\sum_{\boldsymbol{G}'} \epsilon^{-1}(\boldsymbol{G}-\boldsymbol{G}')|\boldsymbol{k}+\boldsymbol{G}'|^2 E(\boldsymbol{G}') = \left(\frac{\omega}{c}\right)^2 E(\boldsymbol{G}). \quad (8)$$

By solving equation (8), general photonic band structures are obtained. Calculational errors between 441 and 729 plane waves are within 1% and, therefore, we calculate with 441 plane waves.

We define the following parameter as the standard of flatness of photonic bands:

$$F = 2\frac{\omega_t - \omega_b}{\omega_t + \omega_b}, \quad (9)$$

where ω_t and ω_b are the top and the bottom of photonic bands, i.e. F is the difference between the top and the bottom relative to the averages of the top and the bottom, and a smaller F value indicates more effective flatness.

3. Numerical calculation and discussion

The photonic band structures formed by electric fields localizing at rods are shown in figure 2. The inset indicates the first Brillouin zone of kagome lattices. Γ, M and K indicate the rotationally symmetric points in the first Brillouin zone. The solid and dotted curves indicate photonic band structures at $\beta'/\beta = 0$ and 0.1, respectively. That is, only the nearest-neighbour interaction is considered in the former, whereas both the nearest- and second-nearest-neighbour

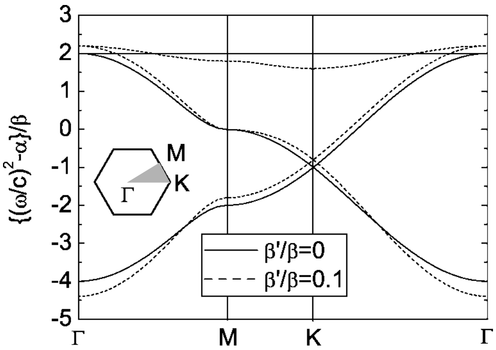

Figure 2. Photonic band structure formed by electric fields localizing at rods. The inset indicates the first Brillouin zone of kagome lattices. Γ, M and K indicate rotationally symmetric points in the first Brillouin zone. The solid and dotted curves indicate photonic band structures at $\beta'/\beta = 0$ and 0.1, respectively.

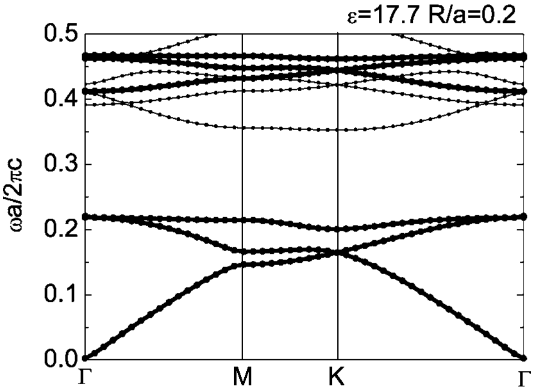

Figure 3. General photonic band structure at $\epsilon = 17.7$ (InSb) and $R/a = 0.2$. Thick lines indicate photonic bands formed by electric fields localizing at rods.

interactions are considered in the latter. As shown in this figure, the third photonic band is a flat photonic band at $\beta'/\beta = 0$, and the flatness of the third photonic band deteriorates with the second-nearest-neighbour interaction. To obtain effective flat photonic bands, therefore, a smaller second-nearest-neighbour interaction is necessary.

Next, we investigate a general photonic band structure by the plane wave method. Figure 3 shows the general photonic band structure at $\epsilon = 17.7$ (InSb) and $R/a = 0.2$. As evident in this figure, the photonic bands drawn by thick curves are similar to those in figure 2, i.e. these photonic bands are formed by electric fields localizing at rods. Two quasi flat photonic bands exist around $\omega a/2\pi c = 0.22$ and 0.46, and we define the former and latter quasi flat photonic bands as the first and second quasi flat photonic bands, respectively. Although the flatness of the first quasi flat photonic band deteriorates at the K point, the second quasi flat photonic band shows an effective flatness.

In figures 4(a)–(d), we show the electric energy distribution of the first and second quasi flat photonic bands to investigate whether these quasi flat photonic bands are formed by electric fields localizing at rods. Figures 4(a) and (b) show the electric energy distributions at the Γ and K points, respectively, in the first quasi flat photonic band, and figures 4(c) and (d)

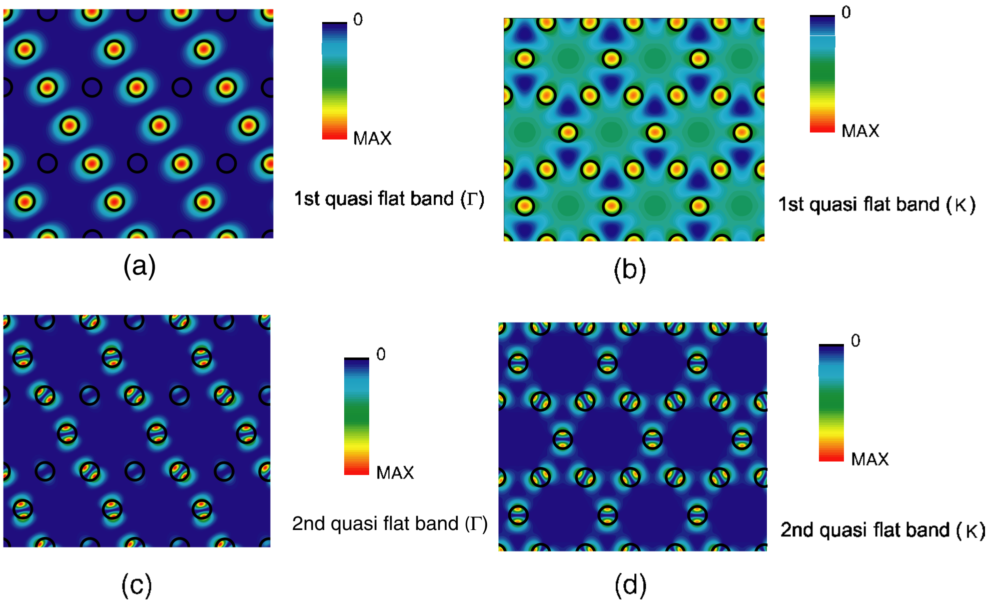

Figure 4. (a), (b) Electric energy distributions of the first quasi flat photonic bands at the Γ and K points, respectively. (c), (d) Electric energy distributions of the second quasi flat photonic bands at the Γ and K points, respectively.

(This figure is in colour only in the electronic version.)

show the electric energy distributions at the Γ and K points, respectively, in the second quasi flat photonic band. As shown in figure 4(a), electric energies localize strongly at rods. In figure 4(b), however, electric energies also exist at points other than the rods, although they become maximum at the rods, i.e. there exists effective interaction other than the nearest-neighbour interaction in the tight-binding model. Therefore, the flatness of the first quasi flat photonic band deteriorates at the K point. In figures 4(c) and (d), on the other hand, the electric energies localize strongly at rods and, therefore, the second quasi flat photonic band shows an effective flatness. That is, the second quasi photonic band is appropriate for the effective enhancement of the omnidirectional electromagnetic interaction of materials because of the electric fields localizing at the materials. The energy distribution of electric fields localizing at the rods is monopolar and dipolar in the first and second quasi flat photonic bands, respectively.

We investigated the dependences of the first and second quasi flat photonic bands on the dielectric constants and radii of rods. Figure 5(a) shows the dependence of F on the dielectric constants of rods at $R/a = 0.2$. In both the first and second quasi flat photonic bands, F decreases monotonically with increasing ϵ. This is because electric localizing effects become strong with increasing dielectric constants of rods, i.e. materials with high dielectric constants such as semiconductors are appropriate for effective flat photonic bands. On the other hand, figure 5(b) shows the dependence of F on radii of rods at $\epsilon = 17.7$. In the first and second quasi flat photonic bands, F becomes minimum around $R/a = 0.2$ and 0.175, respectively. This is because electric fields localizing at larger rods interact with the fields at second-nearest-neighbour rods, while electric localizing effects decrease at smaller rods.

In photonic band structures, there exist uncoupled modes to which plane waves cannot couple due to the antisymmetric electromagnetic distribution for symmetric plane waves. The

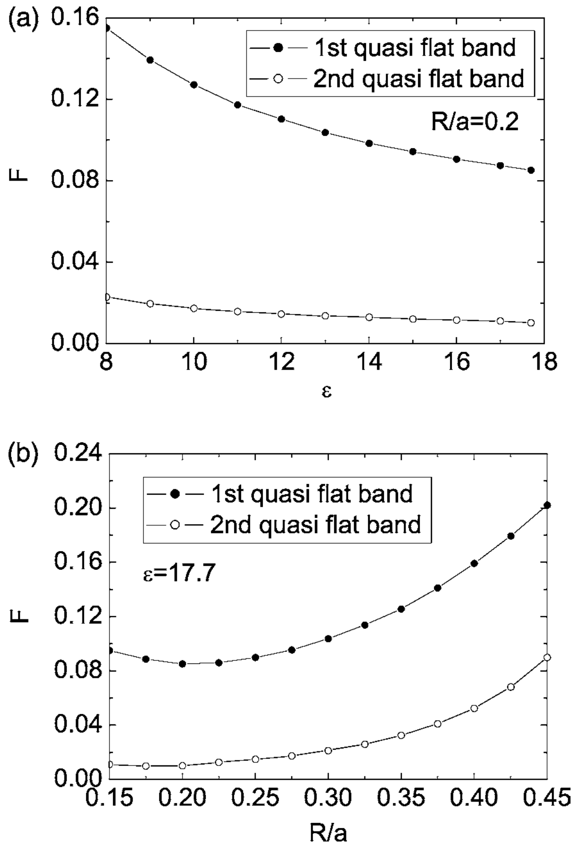

Figure 5. Dependence of F on (a) dielectric constants of rods at $R/a = 0.2$ and (b) radii of rods at $\epsilon = 17.7$.

flat photonic bands considered here are uncoupled modes in the Γ–M and Γ–K directions. However, it has been reported that incident electromagnetic waves without high symmetry can couple to uncoupled modes [18]. Therefore, the flat photonic bands considered here are valid for such incident electromagnetic waves.

With respect to the fabrication of photonic crystals, inverted structures created by drilling holes in backgrounds are appropriate. For the inverted structures shown in figure 1, however, effective flat photonic bands cannot be obtained. This is because electromagnetic waves tend to localize at dielectric materials and, in such structures, electromagnetic waves localizing at the dielectric materials do not form the kagome network.

It has been confirmed that flat photonic bands can be obtained in frequency-dependent materials such as plasmon and polariton [19, 20]. In such materials, however, flat photonic bands appear only in limited frequency ranges. In photonic crystals with kagome lattices, on the other hand, flat photonic bands can be obtained at the desirable frequencies by adjusting lattice constants; moreover, electromagnetic waves satisfying the flat photonic bands always localize at materials, regardless of the directions of the electromagnetic waves. In two-dimensional photonic crystals with kagome lattices, therefore, the omnidirectional lasing and the enhancement of omnidirectional electromagnetic interaction of materials could be achieved.

4. Conclusions

We have theoretically demonstrated the presence of flat photonic bands in two-dimensional photonic crystals composed of circular rods with kagome lattices. The flat photonic bands are formed by electric fields localizing at rods. The energy distribution of the electric fields localizing at rods is monopolar and dipolar in the first and second quasi flat photonic bands, respectively, and the second quasi flat photonic band shows an effective flatness. For flat photonic bands, F decreases with increasing rod dielectric constants and becomes minimum at certain rod radii. Therefore, the two-dimensional photonic crystals with kagome lattices may provide novel applications to the omnidirectional lasing and the enhancement of omnidirectional electromagnetic interaction of materials.

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